

# Pion scalar form factor and model independent values of $f_0(500)$ and $f_0(980)$ meson parameters

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The pion scalar form factor along with all existing S-wave iso-scalar  $\pi\pi$ -scattering phase shift data in the elastic region up to  $1\text{GeV}^2$  are applied for a determination of the  $f_0(500)$  and  $f_0(980)$  meson parameters  $m_{f_0(500)} = (360 \pm 33)\text{MeV}$ ,  $\Gamma_{f_0(500)} = (587 \pm 85)\text{MeV}$  and  $m_{f_0(980)} = (957 \pm 77)\text{MeV}$ ,  $\Gamma_{f_0(980)} = (164 \pm 142)\text{MeV}$  in a model independent way.

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## I. INTRODUCTION

In contrast to other  $SU(3)$  multiplets of hadrons, the identification of the scalar mesons is a long-standing puzzle as some of them have decay widths which cause a strong overlap between resonances and background. The situation below 1 GeV is even more complicated because scalar mesons possess identical quantum numbers with glueballs  $0^{++}$  which can appear just in this mass region.

Despite of this fact, all experimentally established scalar mesons [1] are now classified into light scalar nonet comprising the  $f_0(500)$ ,  $K_0^*(800)$ ,  $f_0(980)$  and  $a_0(980)$  mesons, not necessarily to be  $q\bar{q}$  states, and into regular nonet consisting of the  $f_0(1370)$ ,  $K_0^*(1430)$ ,  $a_0(1450)$  and  $f_0(1500)$  (or  $f_0(1700)$ ) mesons.

It has been difficult to establish the precise  $f_0(500)$  meson parameters because of its large width and because it can certainly not to be determined by a naive Breit-Wigner form. As a result historically this so-called  $\sigma$  meson has been listed in PDG as "not well established" until 1974 and one has believed in the existence of a broad and light scalar iso-scalar resonance.

However, it has been removed from PDG in 1976, because two heavier resonances  $f_0(980)$  and  $f_0(1300)$  were found and  $\sigma$  meson could be replaced by these two heavier ones, which could complete a light ( $q\bar{q}$ ) nonet.

Then again listed back in 1996, after missing more than two decades, although still with an obscure denotation  $f_0(400 - 1200)$ .

From 2002 as "well established"  $f_0(600)$ , but with conservative estimate of the mass 400 – 1200MeV and the width 600 – 1000MeV.

A clarification of this controversial situation has been achieved only recently in the paper [2], where also other recent determinations of reasonable  $f_0(500)$  parameters are presented.

In this paper we confirm the existence of the  $f_0(500)$  scalar meson in a model independent way. As for the latter a representation of the pion scalar form factor is valid in the whole elastic region up to  $1\text{GeV}^2$ , one can determine also the  $f_0(980)$  meson parameters as well.

With this aim, by means of the unitary and analytic approach, an explicit form of the pion scalar form factor (FF) in the language of the absolute value of the pion c.m. three-momentum variable  $q$ , to be connected with the momentum transfer squared by the relation  $t = 4(q^2 + m_\pi^2)$ , is constructed. As a result the pion scalar FF takes the form of a rational function in  $q$ -plane and some of its poles give masses and widths of  $f_0(500)$  and  $f_0(980)$  resonances.

There is a general belief that the  $f_0(500)$  and  $f_0(980)$  mesons are the lowest members of the light scalar nonet, however, it is not definitely clear what other particles are really members of it.

## II. THE PION SCALAR FORM FACTOR

The pion scalar FF  $\Gamma_\pi(t)$  is defined by the matrix element of the quark density

$$\langle \pi^i(p_2) | \hat{m}(\bar{u}u + \bar{d}d) | \pi^j(p_1) \rangle = \delta^{ij} \Gamma_\pi(t) \quad (1)$$

where  $t = (p_2 - p_1)^2$ ,  $\hat{m} = \frac{1}{2}(m_u + m_d)$  and it has similar properties to the pion electromagnetic FF [3].

The pion scalar FF  $\Gamma_\pi(t)$  is analytic in the whole complex  $t$ -plane, except for a cut along the positive real axis, starting at  $t = 4m_\pi^2$ .

For real values  $t < 4m_\pi^2$   $\Gamma_\pi(t)$  is real. The latter implies the so-called reality condition  $\Gamma_\pi^*(t) = \Gamma_\pi(t^*)$ , i.e. that the values of FF above and below the cut are complex conjugate of each other.

At  $t = 0$  the pion scalar FF  $\Gamma_\pi(t)$  coincides with the pion sigma-term [4]  $\Gamma(0) = (0.99 \pm 0.02)m_\pi^2$  to be evaluated in the framework of chiral perturbation theory. Further, the pion scalar FF will be normalized exactly to  $m_\pi^2$ .

The FF  $\Gamma_\pi(t)$  is not directly measurable quantity and it enters e.g. in the matrix element for the decay of the Higgs boson into two pions. However, the contribution to the decay rate seems to be negligible small [5, 6].

If  $\Gamma_\pi(t)$  is evaluated on the upper boundary of the cut, one finds that the following unitarity condition is obeyed

$$Im\Gamma_\pi(t) = \sum_n \langle \pi(p')\pi(p) | T | n \rangle \langle n | \hat{m}(\bar{u}u + \bar{d}d) | 0 \rangle \quad (2)$$

where  $T$  is the  $T$ -operator and the sum runs over a complete set of allowed states  $2\pi, 4\pi, \dots, K\bar{K}, \dots$ , which create additional branch points on the positive real axis of the  $t$ -plane between  $4m_\pi^2$  and  $\infty$ .

In the elastic region  $4m_\pi^2 \leq t \leq 16m_\pi^2$  only the first term on the right hand side of (2) contributes and then

$$Im\Gamma_\pi(t) = \Gamma_\pi(t)(\sigma T_0^0)^* \quad (3)$$

where  $\sigma T_0^0$  is the  $S$ -wave iso-scalar  $\pi\pi$  scattering amplitude

$$M_0^0 = \sigma T_0^0 = \frac{1}{2i}(e^{2i\delta} - 1); \quad (4)$$

$\delta = \delta_0^0 + i\varphi$ ;  $\delta_0^0, \varphi$  real, where  $\delta_0^0$  stands for the  $S$ -wave iso-scalar  $\pi\pi$  phase shift, and  $\varphi > 0$  measures the inelasticity.

Though  $\varphi$  exactly vanishes only below  $t = 16m_\pi^2$ , indeed the phenomenological analysis of the  $\pi\pi$  interactions [7] shows that final states containing more than two particles start playing a significant role only well above  $4m_K^2 \approx 1\text{GeV}^2$ , where the inelastic two-body channel  $\pi\pi \rightarrow K\bar{K}$  opens.

Then

$$Im\Gamma_\pi(t) = \Gamma_\pi(t)e^{-i\delta_0^0}\sin\delta_0^0 \quad (5)$$

for  $4m_\pi^2 \leq t \leq 1\text{GeV}^2$ .

From the relation (5) it follows that the phase  $\delta_\Gamma$  of  $\Gamma_\pi(t)$  coincides with  $\delta_0^0$  and just this identity enables us to obtain the pion scalar FF  $\Gamma_\pi(t)$  behavior valid at the elastic interval  $4m_\pi^2 \leq t \leq 1\text{GeV}^2$  and subsequently allows the identification of the  $f_0(500)$  meson pole and the pole of  $f_0(980)$  meson as well on the second Riemann sheet in  $t$ -variable.

The asymptotic behavior

$$\Gamma_\pi(t)|_{t \rightarrow \infty} \sim \frac{1}{t} \quad (6)$$

is predicted by the quark counting rules.

Starting from the unitarity condition for the  $S$ -wave iso-scalar  $\pi\pi$  scattering amplitude

$$\text{Im}M_0^0 = -|M_0^0|^2 \quad (7)$$

one can do analytic continuation of  $M_0^0$  through the upper and lower boundaries of the unitary elastic cut and to prove in this way the singularity at  $t = 4m_\pi^2$  to be a square root branch point. As a result one gets

$$(M_0^0)^{II} = \frac{(M_0^0)^I}{1 - 2i(M_0^0)^I}. \quad (8)$$

The same can be done with the pion scalar FF and as a result one gets the expression

$$(\Gamma_\pi)^{II} = \frac{(\Gamma_\pi)^I}{1 - 2i(M_0^0)^I}, \quad (9)$$

relating the pion scalar FF on the second Riemann sheet with the pion scalar FF and the  $S$ -wave iso-scalar  $\pi\pi$  scattering amplitude on the first Riemann sheet, demonstrating in this way that the singular point of pion scalar FF at  $t = 4m_\pi^2$  is square root branch point, generating two sheeted Riemann surface on which the pion scalar FF is defined.

Moreover, by a comparison of (8) with (9) one can see that both expressions have identical denominator, from where it automatically follows that if there are  $f_0(500)$  and  $f_0(980)$  mesons in the form of the poles of the  $S$ -wave iso-scalar  $\pi\pi$  scattering amplitude on the second Riemann sheet, then they appear also as poles on the second Riemann sheet of the pion scalar FF.

Now, by an application of the conformal mapping

$$q = [(t - 4)/4]^{1/2}, \quad m_\pi = 1 \quad (10)$$

two-sheeted Riemann surface of  $\Gamma_\pi(t)$  is mapped into one absolute valued pion c.m. three-momentum  $q$ -plane and the elastic cut disappears. Neglecting all higher branch points, there are only poles and zeros of  $\Gamma_\pi(t)$  in  $q$ -plane and as a consequence the pion scalar FF can be represented by a Pad'e-type approximation

$$\Gamma_\pi(t) = \frac{\sum_{n=0}^M a_n q^n}{\prod_{i=1}^N (q - q_i)}. \quad (11)$$

Because  $\Gamma_\pi(t)$  is a real analytic function, the coefficients  $a_n$  in (11) with  $M$  even (odd) are real (pure imaginary), respectively, and the poles  $q_i$  can appear on the imaginary axis or they are placed always two of them symmetrically according to it.

If one multiplies both, the numerator and the denominator of (11), by the complex conjugate factor  $\prod_{i=1}^N (q - q_i)^*$ , the new denominator is a polynomial with real coefficients already and tangent

of the pion scalar FF phase  $\delta_\Gamma(t)$  is given just by the ratio of the imaginary part to the real part of the new numerator as follows

$$\tan \delta_\Gamma(t) = \frac{\text{Im}[\prod_{i=1}^N (q - q_i)^* \sum_{n=1}^M a_n q^n]}{\text{Re}[\prod_{i=1}^N (q - q_i)^* \sum_{n=1}^M a_n q^n]}. \quad (12)$$

Further, by using the identity  $\delta_\Gamma = \delta_0^0$  following from (5) and the threshold behavior of  $\delta_0^0$ , the following parametrization

$$\tan \delta_0^0(t) = \frac{A_1 q + A_3 q^3 + A_5 q^5 + A_7 q^7 + \dots}{1 + A_2 q^2 + A_4 q^4 + A_6 q^6 + \dots} \quad (13)$$

or equivalent relation

$$\delta_0^0(t) = \frac{1}{2i} \ln \frac{(1 + A_2 q^2 + A_4 q^4 + A_6 q^6 + \dots) + i(A_1 q + A_3 q^3 + A_5 q^5 + A_7 q^7 + \dots)}{(1 + A_2 q^2 + A_4 q^4 + A_6 q^6 + \dots) - i(A_1 q + A_3 q^3 + A_5 q^5 + A_7 q^7 + \dots)} \quad (14)$$

is obtained from (12), where  $A_i$  are all real new coefficients. The parameter  $A_1$  is exactly equal to the  $S$ -wave iso-scalar  $\pi\pi$  scattering length  $a_0^0$ .

One can see directly from (13) that if the degree of the numerator is higher than the degree of its denominator then

$$\lim_{q \rightarrow \infty} \delta_0^0(t) = \frac{\pi}{2}. \quad (15)$$

However, if the degree of the numerator in (13) is lower than the degree of its denominator then

$$\lim_{q \rightarrow \infty} \delta_0^0(t) = 0. \quad (16)$$

The above-mentioned asymptotic behaviors can not be solved beforehand and only a comparison of (13) with data on  $\delta_0^0(t)$  can decide what type of pion scalar FF phase representations derived from either the dispersion relation with one subtraction, or from the dispersion relation without subtractions, will be the most suitable in our further considerations.

### III. ANALYSIS OF $S$ -WAVE ISO-SCALAR $\pi\pi$ SCATTERING PHASE SHIFT DATA

There is longdated discussion, what data on  $\delta_0^0(t)$  are more correct, the "down" solution, or the "up" solution. Fortunately, the latter concerns only in data above  $1\text{GeV}^2$ .

As we are interested only for scalar meson resonances below  $1\text{GeV}^2$ , we have collected unambiguous reliable 66 experimental points (see Fig.1) from [8],[9],[10] and [11] at the elastic region without any mutual discrimination and trying to find the best description of them by the

$$\delta_0^0(t) = \arctan \frac{A_1 q + A_3 q^3 + A_5 q^5 + A_7 q^7 + \dots}{1 + A_2 q^2 + A_4 q^4 + A_6 q^6 + \dots} \quad (17)$$

parametrization to be equivalent to (14).

We have carried out the analysis of the data on  $\delta_0^0(t)$  successively, starting with the lowest nonzero coefficient  $A_1$  and then repeating optimal description of the data always adding next coefficient to be different from zero. As a result we analyse the data with one, two, three, etc. parameter expression (17) up to the moment when the minimum of  $\chi^2/ndf$  is achieved.

The results are summarized in the following Table

Number of $A_i$	$\chi^2/ndf$
1	17.75
2	1.66
3	1.60
4	1.49
5	1.41
6	1.44
7	1.50

from where one can see immediately that the minimum of  $\chi^2/ndf$  is achieved with 5 coefficients in (17). They are acquiring the following numerical values

$$\begin{aligned} A_1 &= 0.25684 \pm 0.0107; A_3 = 0.14547 \pm 0.01620; A_5 = -.01217 \pm 0.00070 \\ A_2 &= 0.02274 \pm 0.02830; A_4 = -.01537 \pm 0.00480 \end{aligned} \quad (18)$$

and the description of the data by these coefficients is presented in Fig.1 by full line.

This result (see (15)) is enough to conclude that we have to start construction of the pion scalar FF by the dispersion relation with one subtraction

$$\Gamma_\pi(t) = 1 + \frac{t}{\pi} \int_{4m_\pi^2}^{\infty} \frac{Im\Gamma_\pi(t')}{t'(t' - t)} dt'. \quad (19)$$

#### IV. THE PHASE REPRESENTATION AND EXPLICIT FORM OF THE PION SCALAR FORM FACTOR

Now, substituting the pion scalar FF elastic unitarity condition (5) into the dispersion relation with one subtraction (19) one obtains the so-called Muskhelishvili-Omnes integral equation,

$$\Gamma_\pi(t) = 1 + \frac{t}{\pi} \int_{4m_\pi^2}^{\infty} \frac{\Gamma_\pi(t') e^{-i\delta_0^0} \sin\delta_0^0}{t'(t' - t)} dt', \quad (20)$$

the solution of which is the pion scalar FF phase representation with one subtraction

$$\Gamma_\pi(t) = P_n(t) \exp\left[\frac{t}{\pi} \int_{4m_\pi^2}^{\infty} \frac{\delta_0^0(t')}{t'(t' - t)} dt'\right], \quad (21)$$

where  $P_n(t)$  is an arbitrary polynomial to be restricted with  $P_n(0) = 1$  and its degree must not be higher than  $\delta_0^0(\infty)/\pi$ .

The substitution of  $\delta_0^0(t)$  (17) in the equivalent form (14) with 5 nonzero above-mentioned coefficients into the pion scalar FF phase representation (21) leads to the expression

$$\Gamma_\pi(t) = P_n(t) \exp\left[\frac{(q^2 + 1)}{\pi i} \int_4^\infty \frac{q' \ln \frac{(1+A_2q'^2+A_4q'^4)+i(A_1q'+A_3q'^3+A_5q'^5)}{(1+A_2q'^2+A_4q'^4)-i(A_1q'+A_3q'^3+A_5q'^5)}}{(q'^2 + 1)(q'^2 - q^2)} dq'\right], \quad (22)$$

in which  $m_\pi = 1$  is assumed. Taking into account the fact that the integrand is even function of its argument, i.e. it is invariant under the transformation  $q' \rightarrow -q'$ , the latter expression can be transformed into the following form

$$\Gamma_\pi(t) = P_n(t) \exp\left[\frac{(q^2 + 1)}{2\pi i} \int_{-\infty}^\infty \frac{q' \ln \frac{(1+A_2q'^2+A_4q'^4)+i(A_1q'+A_3q'^3+A_5q'^5)}{(1+A_2q'^2+A_4q'^4)-i(A_1q'+A_3q'^3+A_5q'^5)}}{(q'^2 + 1)(q'^2 - q^2)} dq'\right], \quad (23)$$

where the integral is already suitable to be calculated by means of the theory of residua.

In order to carry out this program one has to identify all poles of the integrand and simultaneously calculate complex roots of the polynomial in the numerator and complex conjugate roots in the denominator under the logarithm, which generate branch points in  $q$ -plane.

Considering the case  $q^2 < 0$  i.e.  $q = i\sqrt{\frac{4-t}{4}} \equiv ib$  one finds the poles of the integrand in  $q' = \pm i$  and  $q' = \pm ib$ .

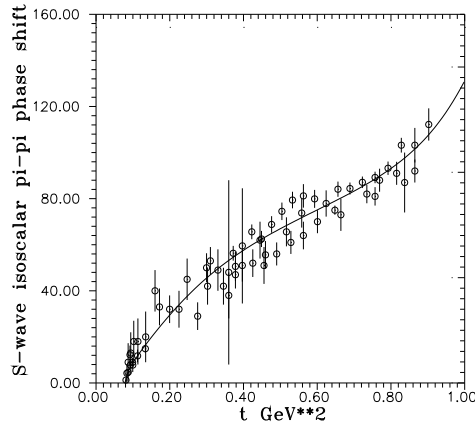


FIG. 1: Description of the S-wave iso-scalar  $\pi\pi$  phase shift by the  $[5/4]$  Pad'e type approximation with the values of parameters (18)

What concerns of the roots of polynomials under the logarithm, it is clear that it is enough to investigate the roots of the numerator as the roots of the denominator are complex conjugate to the roots of the numerator.

So, let us start with an investigation of the numerator  $(1 + A_2q'^2 + A_4q'^4) + i(A_1q' + A_3q'^3 + A_5q'^5) = 0$ .

In order to have equation with real coefficients one substitutes  $q' = ix$ .

Then  $1 - A_1x - A_2x^2 + A_3x^3 + A_4x^4 - A_5x^5 = 0$

or  $-x^5 + \frac{A_4}{A_5}x^4 + \frac{A_3}{A_5}x^3 - \frac{A_2}{A_5}x^2 - \frac{A_1}{A_5}x + \frac{1}{A_5} = 0$

Solutions of the latter equation are the following

$$\begin{aligned} x_1 &= -1.8633297' \\ x_2 &= 0.2832535 - i3.5830748, \\ x_3 &= 1.2800184 - i1.3328447, \\ x_4 &= 0.2832535 + i3.5830748, \\ x_5 &= 1.2800184 + i1.3328447, \end{aligned}$$

from where one finds roots of the numerator and the denominator under the logarithm of integrand  $\phi(q', q)$  to be

$$\begin{aligned} q_1 &= -i1.8633297, \\ q_2 &= -3.5830748 + i0.2832535, \\ q_3 &= -1.3328447 + i1.2800184, \\ q_4 &= 3.5830748 + i0.2832535, \\ q_5 &= 1.3328447 + i1.2800184, \end{aligned} \tag{24}$$

and

$$\begin{aligned} q_1^* &= -q_1, \\ q_2^* &= -q_4, \\ q_3^* &= -q_5, \\ q_4^* &= -q_2, \\ q_5^* &= -q_3, \end{aligned} \tag{25}$$



respectively.

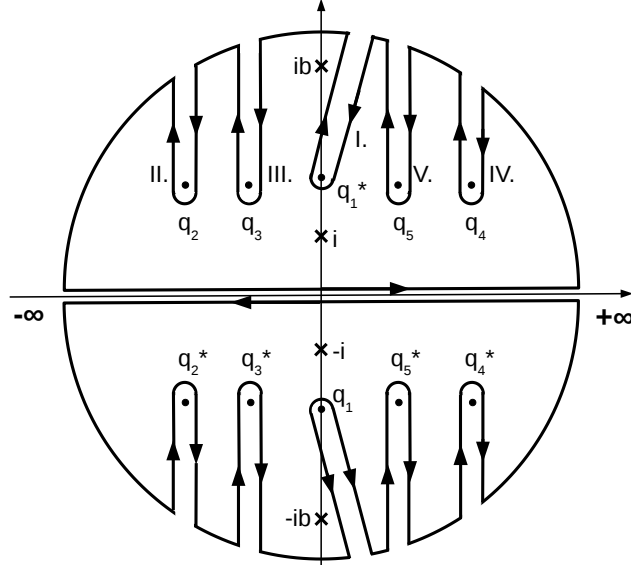


FIG. 2: Poles ( $\times$ ) and branch points ( $\bullet$ ) of the integrands  $\phi_1(q', q)$  and  $\phi_2(q', q)$  with contours of integrations in the upper and the lower half-planes, respectively.

Then the integral in (23) takes the form

$$I = \int_{-\infty}^{\infty} \frac{q' \ln \frac{(q' - q_1)(q' - q_2)(q' - q_3)(q' - q_4)(q' - q_5)}{(q' - q_1^*)(q' - q_2^*)(q' - q_3^*)(q' - q_4^*)(q' - q_5^*)}}{(q' + i)(q' - i)(q' + ib)(q' - ib)} dq', \quad (26)$$

with all singularities of its integrand to be explicitly presented in Fig.2.

For an explicit calculation of the latter integral (26) it is convenient to split it into sum of two integrals

$$I = \int_{-\infty}^{\infty} \frac{q' \ln \frac{(q' - q_2)(q' - q_3)(q' - q_4)(q' - q_5)}{(q' - q_1^*)}}{(q' + i)(q' - i)(q' + ib)(q' - ib)} dq' + \\ + \int_{-\infty}^{\infty} \frac{q' \ln \frac{(q' - q_1)}{(q' - q_2^*)(q' - q_3^*)(q' - q_4^*)(q' - q_5^*)}}{(q' + i)(q' - i)(q' + ib)(q' - ib)} dq' = I_1 + I_2$$

according to singularities to be placed in the upper or lower half-plane, respectively.

Let us start to calculate the first integral  $I_1$  by the theory of residua

$$\oint \frac{q' \ln \frac{(q' - q_2)(q' - q_3)(q' - q_4)(q' - q_5)}{(q' - q_1^*)}}{(q' + i)(q' - i)(q' + ib)(q' - ib)} dq' = 2\pi i \sum_{n=1}^2 \text{Res}_n \quad (27)$$

where the contour of integration is closed in the upper half-plane (see Fig.2).

As the integral on the half-circle is 0 then

$$I_1 = \int_{-\infty}^{\infty} \phi_1(q') dq' = 2\pi i \sum_{n=1}^2 Res_n - \left[ -\int_{1^*} + \int_2 + \int_3 + \int_4 + \int_5 \right] \quad (28)$$

where the integrals on the right-hand side represent contributions of the cuts generated by the branch points  $q_1^*, q_2, q_3, q_4, q_5$  in Fig.2.

The residua at the poles  $q' = i, q' = ib$  are straightforward to calculate and they are

$$Res\phi_1(i, q) = -\frac{1}{2(q^2 + 1)} \ln \frac{(i - q_2)(i - q_3)(i - q_4)(i - q_5)}{(i - q_1^*)}, \quad (29)$$

$$Res\phi_1(ib, q) = \frac{1}{2(q^2 + 1)} \ln \frac{(q - q_2)(q - q_3)(q - q_4)(q - q_5)}{(q - q_1^*)}, \quad (30)$$

as  $ib = q$ .

Now the contributions of the cuts. Let us start with the contribution of the cut to be generated by the branch point  $q_1^*$ .

$$\begin{aligned} \int_{1^*} &= \int_{\infty}^{q_1^*} \frac{q' \ln_+(q' - q_1^*)}{(q'^2 + 1)(q'^2 + b^2)} dq' + \int_{q_1^*}^{\infty} \frac{q' \ln_-(q' - q_1^*)}{(q'^2 + 1)(q'^2 + b^2)} dq' = \\ &= \int_{q_1^*} \frac{q'}{(q'^2 + 1)(q'^2 + b^2)} [\ln_-(q' - q_1^*) - \ln_+(q' - q_1^*)] dq' = \\ &= -2\pi i \int_{q_1^*} \frac{q'}{(q'^2 + 1)(q'^2 + b^2)} dq' = \\ &= -\frac{\pi i}{(b^2 - 1)} \ln \frac{(q_1^{*2} + b^2)}{(q_1^{*2} + 1)} \equiv \frac{1}{2} \frac{2\pi i}{(q^2 + 1)} \ln \frac{(q_1^{*2} - q^2)}{(q_1^{*2} + 1)}. \end{aligned} \quad (31)$$

Similarly

$$\int_j = -\frac{\pi i}{(b^2 - 1)} \ln \frac{(q_j^2 + b^2)}{(q_j^2 + 1)} \equiv \frac{1}{2} \frac{2\pi i}{(q^2 + 1)} \ln \frac{(q_j^2 - q^2)}{(q_j^2 + 1)}; \quad j = 2, 3, 4, 5. \quad (32)$$

Then the sum of all these partial results according to (28) gives the final result for  $I_1$  in the form

$$I_1 = \frac{1}{2} \frac{2\pi i}{(q^2 + 1)} \ln \frac{(q + q_1^*)}{(q + q_2)(q + q_3)(q + q_4)(q + q_5)} \frac{(i + q_2)(i + q_3)(i + q_4)(i + q_5)}{(i + q_1^*)}. \quad (33)$$

Similarly one can calculate also the second integral  $I_2$  by means of the theory of residua

$$\oint \frac{q' \ln \frac{(q' - q_1)}{(q' - q_2^*)(q' - q_3^*)(q' - q_4^*)(q' - q_5^*)}}{(q' + i)(q' - i)(q' + ib)(q' - ib)} dq' = 2\pi i \sum_{n=1}^2 Res_n \quad (34)$$

where the contour of integration is closed in the lower half-plane (see Fig.2).

As the integral on the half-circle is 0 then

$$I_2 = \int_{-\infty}^{\infty} \phi_2(q') dq' = -2\pi i \sum_{n=1}^2 Res_n + \left[ + \int_1 - \int_{2^*} - \int_{3^*} - \int_{4^*} - \int_{5^*} \right]. \quad (35)$$

The residua at the poles  $q' = -i, q' = -ib$  take the form

$$Res\phi_2(-i, q) = -\frac{1}{2(q^2 + 1)} \ln \frac{(-i - q_1)}{(-i - q_2^*)(-i - q_3^*)(-i - q_4^*)(-i - q_5^*)}, \quad (36)$$

$$Res\phi_2(-ib, q) = \frac{1}{2(q^2 + 1)} \ln \frac{(-q - q_1)}{(-q - q_2^*)(-q - q_3^*)(-q - q_4^*)(-q - q_5^*)}, \quad (37)$$

as  $ib = q$ .

The contribution of the cut to be generated by the branch point  $q_1$  is

$$\begin{aligned} \int_1 &= \int_{\infty}^{q_1} \frac{q' \ln_+(q' - q_1)}{(q'^2 + 1)(q'^2 + b^2)} dq' + \int_{q_1}^{\infty} \frac{q' \ln_-(q' - q_1)}{(q'^2 + 1)(q'^2 + b^2)} dq' = \\ &= \int_{q_1}^{\infty} \frac{q'}{(q'^2 + 1)(q'^2 + b^2)} [\ln_-(q' - q_1) - \ln_+(q' - q_1)] dq' = \\ &= -2\pi i \int_{q_1}^{\infty} \frac{q'}{(q'^2 + 1)(q'^2 + b^2)} dq' = \\ &= -\frac{\pi i}{(b^2 - 1)} \ln \frac{(q_1^2 + b^2)}{(q_1^2 + 1)} \equiv \frac{1}{2} \frac{2\pi i}{(q^2 + 1)} \ln \frac{(q_1^2 - q^2)}{(q_1^2 + 1)}. \end{aligned} \quad (38)$$

Similarly

$$\int_{j^*} = -\frac{\pi i}{(b^2 - 1)} \ln \frac{(q_j^{*2} + b^2)}{(q_j^{*2} + 1)} \equiv \frac{1}{2} \frac{2\pi i}{(q^2 + 1)} \ln \frac{(q_j^{*2} - q^2)}{(q_j^{*2} + 1)}; \quad j = 2, 3, 4, 5. \quad (39)$$

Then the sum of all these partial results according to (35) gives the comprehensive result for  $I_2$

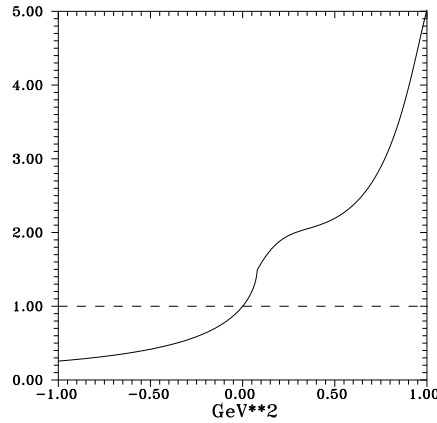


FIG. 3: Behavior of the pion scalar form factor in the region  $-1\text{GeV}^2 < t < 1\text{GeV}^2$

in the form

$$I_2 = \frac{1}{2} \frac{2\pi i}{(q^2 + 1)} \left[ \ln \frac{(q + q_1)}{(q + q_2^*)(q + q_3^*)(q + q_4^*)(q + q_5^*)} \frac{(i + q_2^*)(i + q_3^*)(i + q_4^*)(i + q_5^*)}{(i + q_1)} + \right. \\ \left. + \ln \frac{q_1^2 - q^2}{q_1^2 + 1} - \frac{q_2^{*2} - q^2}{q_2^{*2} + 1} - \frac{q_3^{*2} - q^2}{q_3^{*2} + 1} - \frac{q_4^{*2} - q^2}{q_4^{*2} + 1} - \frac{q_5^{*2} - q^2}{q_5^{*2} + 1} \right] \quad (40)$$

from where by using the relations (25) finally one gets

$$I_2 = \frac{1}{2} \frac{2\pi i}{(q^2 + 1)} \ln \frac{(q + q_1^*)}{(q + q_2)(q + q_3)(q + q_4)(q + q_5)} \frac{(i + q_2)(i + q_3)(i + q_4)(i + q_5)}{(i + q_1^*)}. \quad (41)$$

The sum of (41) with (33) represents the total integral

$$I = \frac{2\pi i}{(q^2 + 1)} \ln \frac{(q - q_1)}{(q + q_2)(q + q_3)(q + q_4)(q + q_5)} \frac{(i + q_2)(i + q_3)(i + q_4)(i + q_5)}{(i - q_1)}. \quad (42)$$

If the latter is substituted into the pion scalar FF phase representation (23) one obtains an explicit form for the pion scalar FF  $\Gamma_\pi(t)$

$$\Gamma_\pi(t) = P_n(t) \frac{(q - q_1)}{(q + q_2)(q + q_3)(q + q_4)(q + q_5)} \frac{(i + q_2)(i + q_3)(i + q_4)(i + q_5)}{(i - q_1)}, \quad (43)$$

which behavior graphically is presented in Fig.3.

The  $-q_3$  and  $-q_2$  poles of  $\Gamma_\pi(t)$  on the second Riemann sheet in  $t$ -variable correspond to  $f_0(500)$  and  $f_0(980)$  scalar meson resonances, respectively.

Their masses and widths are determined to be

$$\mathbf{m}_{f_0(500)} = (\mathbf{360} \pm \mathbf{33})\text{MeV}, \quad \mathbf{\Gamma}_{f_0(500)} = (\mathbf{587} \pm \mathbf{85})\text{MeV},$$

$$\mathbf{m}_{f_0(980)} = (\mathbf{957} \pm \mathbf{72})\text{MeV}, \quad \mathbf{\Gamma}_{f_0(980)} = (\mathbf{164} \pm \mathbf{142})\text{MeV},$$

where the errors correspond to the transferred errors of the coefficients of (18).

The parameters of  $f_0(500)$  can be compared with other determinations presented in the following Table

$$m_\sigma = 441\text{MeV}, \quad \Gamma_\sigma = 544\text{MeV}, [12]$$

$$m_\sigma = 474\text{MeV}, \quad \Gamma_\sigma = 508\text{MeV}, [13]$$

$$m_\sigma = 463\text{MeV}, \quad \Gamma_\sigma = 508\text{MeV}, [14]$$

$$m_\sigma = 443\text{MeV}, \quad \Gamma_\sigma = 432\text{MeV}, [15]$$

$$m_\sigma = 452\text{MeV}, \quad \Gamma_\sigma = 520\text{MeV}, [16]$$

$$m_\sigma = 453\text{MeV}, \quad \Gamma_\sigma = 542\text{MeV}, [17]$$

$$m_\sigma = 457\text{MeV}, \quad \Gamma_\sigma = 558\text{MeV}, [2].$$

As one can see immediately from this Table, its mass in these determinations is slightly higher, whereas the width is lower, than in our model independent approach.

Finally one can only say, that if more precise data on the S-wave iso-scalar  $\pi\pi$  scattering phase shift is available, more precise parameters of  $f_0(500)$  and  $f_0(980)$  can be found in our model independent method of their determination.

## V. CONCLUSIONS

The unitary and analytic approach has been applied for a prediction of the pion scalar FF behavior in elastic region, in the framework of which only the experimental data on S-wave iso-scalar  $\pi\pi$  scattering phase shift in elastic region were used to determine the  $f_0(500)$  and  $f_0(980)$  scalar meson parameters in a model independent way.

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